Appendix C

Theoretical Considerations

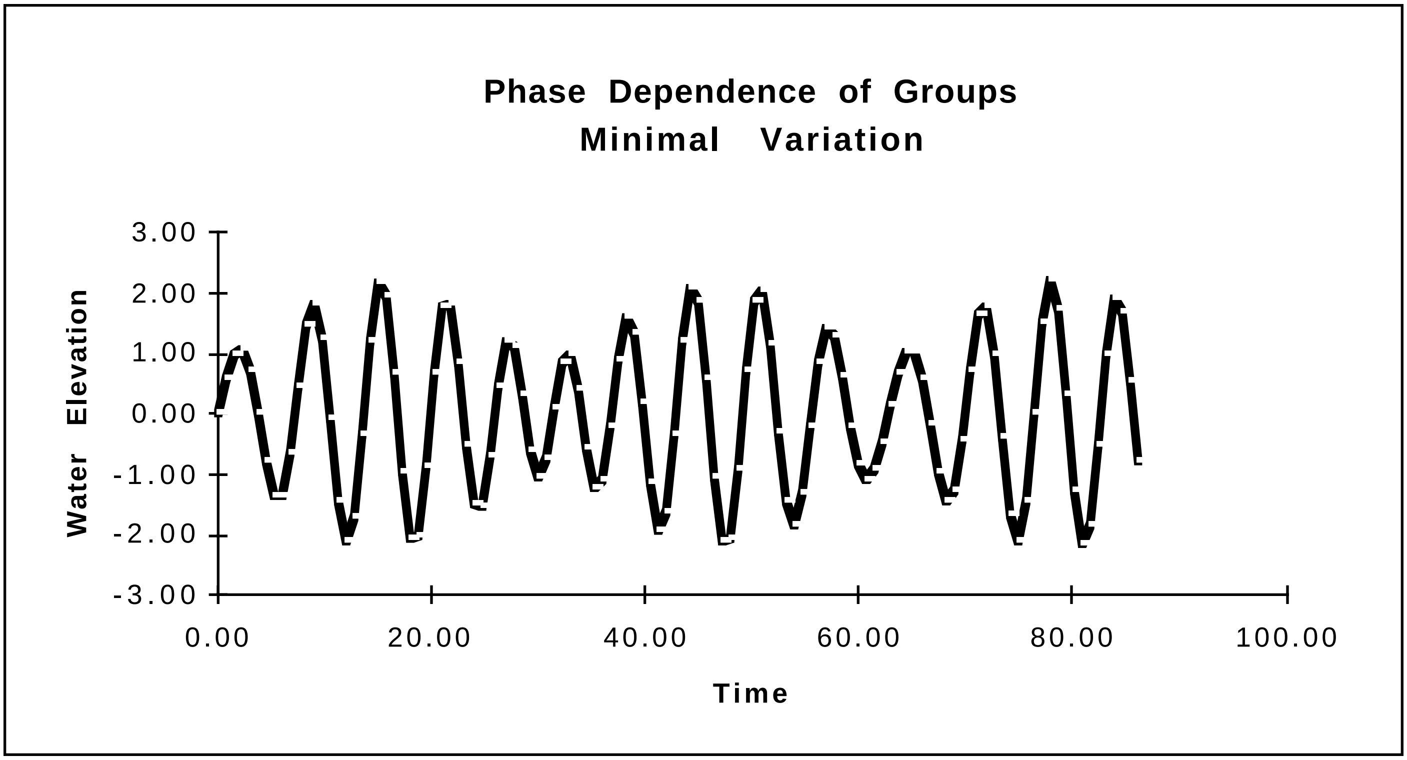
Thinking in Terms of Wave Groups; Wave-Basin Caveats

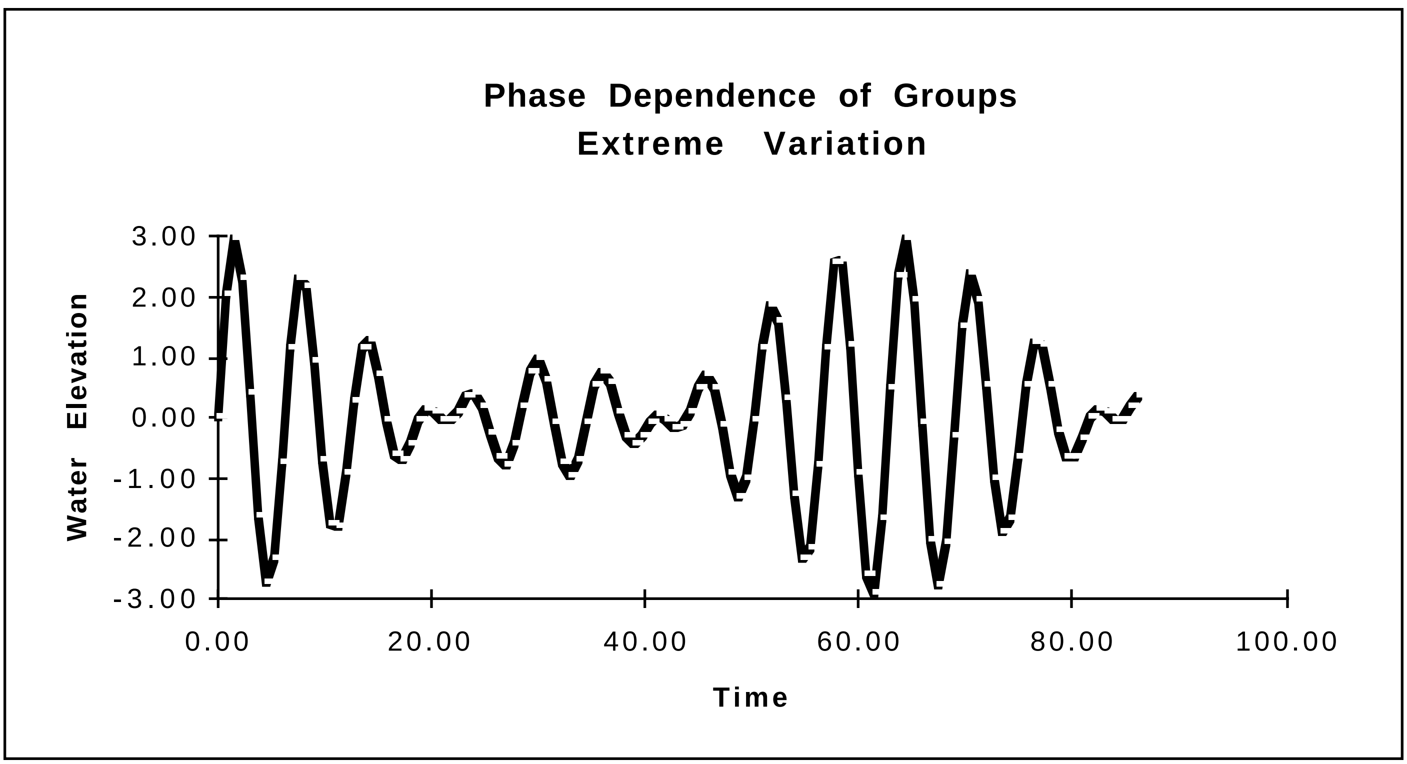
The "low-frequency wave spectral density" responsible for wave-induced long-period oscillations of a moored vessel can be thought of simply as the spectrum of the *envelope* of the waves, or equivalently as the spectrum of wave *groups.* It is extremely important to realize that it is mathematically (and physically) impossible to define without ambiguity the group spectrum from a specified wave spectrum. This fact is illustrated below. The spectral values produced in Output Section IV by Slowsim relate to a "perfectly random" phase relationship between the regular wave components comprising the irregular wave train. Because a physically realizable (but negligibly probable in nature) phase map for the component waves can produce a vastly different low-frequency spectral density than the one predicted by Slowsim, much care must be taken when comparing simulation results with wave-basin measurements. Because all wave basins are designed by humans, and humans often have difficulty producing truly random inputs, the group spectra produced in a wave basin can be horribly unrealistic and can result in vessel low-frequency motions and mooring loads substantially larger or smaller than will be realized in "nature's laboratory".

This problem is similar to the familiar "statistical paradox" that the Second Law of Thermodynamics does not really speak to physical *impossibilities*, but rather to hopelessly remote *improbabilities.* For example, no physical law denies the formal possibility that at some moment all the air will rush from the room in which you sit, creating for an instant a perfect vacuum (and producing a moment of discomfort). Such events, though physically possible, are so fantastically unlikely that no human or any other creature anywhere is likely to witness such an occurrence in the entire history of the universe. However, by careful control of the phase of individual regular wave components in an irregular wave field, we *can* produce group spectra that are hopelessly improbable in nature.

Simple Three-Wave Demonstration

A simple composite "sea" consisting of three regular waves of slightly different period and equal amplitudes illustrates qualitatively (and clearly) the problem. We plot below the water surface elevation for the three wave composite using two different phase relationships between the component waves, one *favorable* for "groupiness", the other *unfavorable*. The difference is readily apparent; these two examples have the *same* "spectrum" and the *same* "wave energy" or "significant wave height" or more accurately, the same wave variance. This phase freedom and its consequent effect on group spectra applies to *all* wave spectra regardless how complex.


Phase Dependence of Groups
Minimal Variation
3.00
2.00
1.00
0.00
-1.00
-2.00
-3.00
Water Elevation
0.00
20.00
40.00
60.00
80.00
100.00
Time


Phase Dependence of Groups
Extreme Variation
3.00
2.00
1.00
0.00
-1.00
-2.00
-3.00
Water Elevation
0.00
20.00
40.00
60.00
80.00
100.00
Time